1 Notes on Construction of Cremers et al factors

Cremers et al constructs two factors:

1. **JUMP factor** consists of two straddles: one expiring in the next calendar month and one expiring in a month after that. Both straddles should be market neutral, i.e. have a beta of zero. To do this we need to weight call and put in a straddle accordingly. Let the weight of the call in a given straddle be θ, then

$$\theta\beta_c + (1-\theta)\beta_p = 0 \implies \theta = \frac{\beta_p}{\beta_p - \beta_c}$$

Once we picked θ for each of the straddles, we need to go long one unit of short-horizon straddle and short y units of long-horizon straddles. We need to pick y so that *vega* of the strategy is *zero*. First note that the vegas of calls and puts are the same and are given by

$$Se^{-q\tau}\phi(d_1)\sqrt{\tau} \text{ where } d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)\,\tau}{\sigma\sqrt{\tau}}$$

Therefore, the vega of each straddle is given by vega of either option. Once we calculated vegas of each straddle we weight them so that

$$1 \cdot vega(\text{straddle 1}) - y \cdot vega(\text{straddle 2}) = 0 \implies y = \frac{vega(\text{straddle 1})}{vega(\text{straddle 2})}$$

In the end, we have a portfolio consisting of four options:

Option	weight
Short horizon call	θ_1
Short horizon put	$1-\theta_1$
Long horizon call	$y \cdot \theta_2$
Long horizon put	$y \cdot (1-\theta_2)$

We hold this portfolio for one day and sell it the next trading day

2. **VOL factor** is almost exactly the same, we also construct a two market neutral straddles with with different maturities. Then, we take a long one unit position in the longer dated straddle and short y units of shorter dated straddle. We pick y to set *gamma* of the strategy to be equal to zero:

$$y = \frac{gamma(\text{straddle 2})}{gamma(\text{straddle 1})}$$

How to calculate betas? Authors do it according to Black-Scholes world where the price follows a geometric Brownian motion. There the evolution of Call price is given by Ito's Lemma as

$$dC = \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 dt + \frac{\partial C}{\partial t}dt$$

Instantaneous covariance of the return on the call and return on the market is given by

$$cov\left(\frac{dC}{C},\frac{dS_m}{S_m}\right) = cov\left(\frac{\partial C}{\partial S}\frac{1}{C}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\frac{1}{C}dt + \frac{\partial C}{\partial t}\frac{1}{C}dt, \frac{dS_m}{S_m}\right) =$$

$$= cov\left(\frac{\partial C}{\partial S}\frac{S}{C}\frac{dS}{S},\frac{dS_m}{S_m}\right) = \Delta\frac{S}{C}cov\left(\frac{dS}{S},\frac{dS_m}{S_m}\right)$$

which implies the following instantaneous beta relationship

$$\boxed{\beta_c = \Delta\frac{S}{C}\cdot\beta_s}$$

To get the beta of the put we can use the put call parity. Ignore all parts that are not stochastic as they will drop from covariances:

$$C = P + S + \dots$$

$$dC = dP + dS + \dots$$
$$\frac{dC}{C} = \frac{dP}{C} + \frac{dS}{C} + \dots$$
$$\frac{dC}{C} = \frac{P}{C}\frac{dP}{P} + \frac{S}{C}\frac{dS}{S} + \dots$$

which implies the following beta relationship

$$\beta_c = \frac{P}{C}\cdot\beta_p + \frac{S}{C}\cdot\beta_s \implies \boxed{\beta_p = \frac{C}{P}\cdot\beta_c - \frac{S}{P}\cdot\beta_s}$$

Since the authors work with index options they set $\beta_s = 1$ which allows to calculate $\beta_c \implies \beta_p \implies \theta$ using observables.